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Simulation of quantum logic via collisions of vector solitons

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Abstract
The polarization of a multi-component vector soliton (of the Manakov type) can be thought of as a state vector of a system of qubits (a register of quantum information). A change of this state on demand via colliding the register pulse with other solitons is shown to be possible with arbitrary accuracy. The parameters (polarization, pulse width, velocity) of the register-switching solitons corresponding to the computationally universal set of quantum gates are found. Physical realizations of information processing using effects of the self-focusing of optical media or of the self-induced transparency are considered.

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1. Introduction

Different authors have proposed using vector solitons as carriers of information which can be switched via collisions with other vector solitons [1–4]. In particular, a concept of computation with Manakov solitons [5] has been developed in [3], and a computationally universal set of (classical) logical gates has been designed for this aim by Steiglitz [6]. Vector solitons of the Manakov type seem to be naturally useful as carriers of quantum information since they are characterized by a polarization (unit) vector of complex components which can be thought of as a multi-qubit state vector. The fast decoherence unavoidable for quantum (microscopic) systems which is the main problem for practical implementation of the quantum logic is expected to be reduced for such mesoscopic objects as solitons. However, the difficulty of quantum information processing with the Manakov solitons results from the nonlinearity of the transformation of their polarization vectors due to the collision.

The present paper deals with the possibility of simulating the quantum logical gates via collisions of the vector solitons of the Manakov type. The accuracy of the considered operations is found to be determined by the method of soliton creation and measuring their polarization; thus, its limitations are not of a fundamental character. I find the parameters of solitons which collide with a register pulse acting as quantum logical gates of the computationally universal set [7]: *CNOT*, *Hadamard*, $\pi/8$, and *phase*. However, the number of collisions necessary to perform these operations depends on the size of the state space (it grows exponentially with the number of qubits). Thus, the presented method is not called the realization of the quantum logical gates but just their simulation. It enables one to build classical counterparts of simple (of a small qubit number) quantum circuits. The advantage results from the fact that it does not demand the use of signal converters (nor amplifiers) in order to perform a sequence of universal operations with solitons in contrast e.g. to the method of [6].

The information processing applications of the vector solitons are based on the solution of an inverse scattering problem by Manakov which was originally formulated as a step of the solution of the two-component nonlinear Schrodinger (NLS) equation describing the self-focusing (SF) of optical media. However, a similar inverse problem appears for other systems. In particular, the self-induced transparency (SIT) of a three-level medium for the so-called Λ or V configurations of the atom–photon coupling is described with similar equations as those decomposing the two-component NLS equation [8]. Thus, the scheme of the soliton-polarization switching via the pulse collisions is common for different physical realizations.

Section 2 is devoted to an outline of possible realizations of vector solitons of the Manakov type and their collisions. In section 3, a construction of universal quantum gates using vector-soliton collisions is carried out. Conclusions are given in section 4.

2. Basics of vector solitons

I outline realizations of the multi-component vector solitons in SF media and media displaying SIT. Let us study the pulse equations of motion and their one-soliton solutions as well as asymptotic two-(colliding)-soliton solutions.

2.1. Multi-component SF of nonlinear optical media

The envelope of a monochromatic electro-magnetic wave propagating in a nonlinear (Kerr) medium is described with the two-component NLS equation. It was solved by Manakov within the inverse scattering [9] approach [5], and the propagation of vector solitons predicted by him has been observed e.g. in optical fibres [10], semiconductor waveguides [11], photorefractive crystals [12]. Considering the propagation of an N-chromatic pulse, one uses the $2N$-component NLS equation describing the motion of the envelopes of the pulse components

$$\mathrm{i}\epsilon_{j,\tau} + \epsilon_{j,xx} + \frac{1}{2}\sum_{k=1}^{2N}|\epsilon_k|^2\epsilon_j = 0. \tag{1}$$

Here, $A_{,\tau} \equiv \partial A/\partial\tau$ and $A_{,x} \equiv \partial A/\partial x$ denote differentials over renormalized time and distance variables. The one-soliton solution of (1) takes the form

$$\epsilon_j(x,\tau) = 4\mathrm{i}c_j\zeta'' \exp[\mathrm{i}2\zeta' x + \mathrm{i}4(\zeta'^2 - \zeta''^2)\tau]\,\mathrm{sech}[2\zeta''(x-x_0) + 8\zeta'\zeta''\tau], \tag{2}$$

where c_j denote components of a (complex) polarization vector of unit length. The constant ζ such that $\zeta' \equiv \mathrm{Re}\,\zeta$, $\zeta'' \equiv \mathrm{Im}\,\zeta$ is called a (complex) wavenumber.

2.2. Multi-component SIT

Let us consider an $N+1$-level atomic medium coupled to an N-component optical pulse in such a way that the lowest atomic level is linearly coupled to others (a V-bouquet configuration of [8], see also [13]). The pulse propagation is described with Maxwell–Bloch equations, and the slowly varying envelope approximation is used for decomposition of the Maxwell wave equation (see e.g. [14]). Let us define the electronic-level occupation amplitudes b, a_j (state-vector components) for the ground and excited levels, respectively, that satisfy $\sum_{j=1}^{N}|a_j|^2 + |b|^2 = 1$, and a spectral distribution $g(\alpha)$ characterizing the inhomogeneous broadening of the medium and normalized to unity, $\int_{-\infty}^{\infty} g(\alpha)\,\mathrm{d}\alpha = 1$. Following [15, 16], the evolution of the density-matrix components $\lambda_{ij} \equiv 2a_i a_j^*$, $\lambda_{j0} \equiv 2a_j b^*$, $\lambda_{00} \equiv 2bb^*$ and of the envelopes of the pulse components ϵ_j is described with

$$\epsilon_{j,x} = \langle \lambda_{j0} \rangle, \tag{3a}$$

$$\lambda_{j0,\tau} + 2\mathrm{i}\zeta' \lambda_{j0} = \tfrac{1}{2}\epsilon_j \lambda_{00} - \tfrac{1}{2}\epsilon_k \lambda_{jk}, \tag{3b}$$

$$\lambda_{ij,\tau} = \tfrac{1}{2}\epsilon_i \lambda_{j0}^* + \tfrac{1}{2}\epsilon_j^* \lambda_{i0}, \tag{3c}$$

$$\lambda_{00,\tau} = -\tfrac{1}{2}\epsilon_k \lambda_{k0}^* - \tfrac{1}{2}\epsilon_k^* \lambda_{k0}. \tag{3d}$$

Here $\langle A \rangle = \int_{-\infty}^{\infty} A(\alpha) g(\alpha)\,\mathrm{d}\alpha$, and a frequency detuning ζ' is assumed to be the same for all the coupled electro-magnetic modes. Equations (3b)–(3d) are equivalent to

$$a_{j,\tau} + \mathrm{i}\zeta' a_j = \tfrac{1}{2}\epsilon_j b, \qquad b_{,\tau} - \mathrm{i}\zeta' b = -\tfrac{1}{2}\epsilon_k^* a_k. \tag{4}$$

Solving the system of (3a) and (4) with the inverse scattering method, one finds the one-soliton scattering potentials

$$\epsilon_j(x,\tau) = 4c_j\zeta'' \exp[\mathrm{i}\omega' x + \mathrm{i}2\zeta'\tau]\,\mathrm{sech}[\omega''(x - x_0) + 2\zeta''\tau], \tag{5}$$

for $a_j = c_j a$ and $\sum_{j=1}^{N} |c_j|^2 = 1$. Here, $\omega' + \mathrm{i}\omega'' = -\frac{1}{2}\int_{-\infty}^{\infty} \frac{g(\alpha)\mathrm{d}\alpha}{\zeta - \alpha}$.

2.3. Soliton collisions

Following Manakov, the collision of two vector solitons characterized by the polarizations $\boldsymbol{c}_{(1)}, \boldsymbol{c}_{(2)}$ and by the wavenumbers ζ_1, ζ_2 respectively results only in the change of the polarization of both solitons according to the transformation

$$\begin{aligned} \boldsymbol{c}'_{(1)} &= \frac{1}{\chi}\frac{\zeta_1^* - \zeta_2}{\zeta_1^* - \zeta_2^*}\left[\boldsymbol{c}_{(1)} + \frac{\zeta_2 - \zeta_2^*}{\zeta_2^* - \zeta_1^*}(\boldsymbol{c}_{(2)}^* \cdot \boldsymbol{c}_{(1)})\boldsymbol{c}_{(2)}\right] \\ \boldsymbol{c}'_{(2)} &= \frac{1}{\chi}\frac{\zeta_1^* - \zeta_2}{\zeta_1 - \zeta_2}\left[\boldsymbol{c}_{(2)} + \frac{\zeta_1 - \zeta_1^*}{\zeta_2 - \zeta_1}(\boldsymbol{c}_{(1)}^* \cdot \boldsymbol{c}_{(2)})\boldsymbol{c}_{(1)}\right], \end{aligned} \tag{6}$$

where

$$\chi \equiv \chi(\boldsymbol{c}_{(1)}, \boldsymbol{c}_{(2)}) = \frac{|\zeta_1 - \zeta_2^*|}{|\zeta_1 - \zeta_2|}\left[1 + \frac{(\zeta_1 - \zeta_1^*)(\zeta_2^* - \zeta_2)}{|\zeta_1 - \zeta_2|^2}|\boldsymbol{c}_{(1)}^* \cdot \boldsymbol{c}_{(2)}|^2\right]^{1/2}. \tag{7}$$

The wavenumbers do not change due to the collision. The above transformation is relevant for the collisions of the SF pulses as well as of the SIT pulses since the inverse scattering equations (of the form of (4)) are similar for both cases (see [5] and [8] for comparison).

3. Quantum logical gates

We consider a 2^n-component vector soliton of the wavenumber ζ as an n-qubit register (in nomenclature of [17], such a classical object is called an n-cebit register). Its polarization (state) vector is transformed after collision with another soliton (called a switching soliton). Let us study the possibility of finding the switching solitons whose collisions with the register soliton change its state vector as the quantum gates of the universal set: *CNOT*, *Hadamard*, $\pi/8$. The fourth gate of the universal set, *phase*, is known to be the composition of two

$\pi/8$-gates. The parameters of the switching solitons, sequences of which correspond to the quantum gates; the polarizations $\boldsymbol{c}_y, \boldsymbol{d}_y, \ldots$ and wavenumbers $\zeta_y, \eta_y, \ldots$ are indexed with $y = a, b, \ldots$. These indices relate to a register qubit which is switched via the collision. We assign consecutive qubits $a, b, \ldots$ to the components $c_1, c_2, \ldots$ of the register polarization following the scheme presented below for a three-qubit case. A state of the system of qubits a, b, c corresponding to the lower, middle and upper wires of quantum circuits respectively is written with the vector

$$|\boldsymbol{c}\rangle = c_1|0\rangle_c|0\rangle_b|0\rangle_a + c_2|0\rangle_c|0\rangle_b|1\rangle_a + c_3|0\rangle_c|1\rangle_b|0\rangle_a + c_4|0\rangle_c|1\rangle_b|1\rangle_a + c_5|1\rangle_c|0\rangle_b|0\rangle_a + c_6|1\rangle_c|0\rangle_b|1\rangle_a + c_7|1\rangle_c|1\rangle_b|0\rangle_a + c_8|1\rangle_c|1\rangle_b|1\rangle_a. \quad (8)$$

Due to the collision, a state vector of the register $\boldsymbol{c}$ transforms into $L(\boldsymbol{c}_y)\boldsymbol{c}$ following (6).

Since all the logical gates are connected to linear transformations of the state vector while the transformation $L(\boldsymbol{c}_y)$ is not linear in general, we search for which parameters of the switching soliton it becomes so. Let us define a linear transformation $\mathcal{L}(\boldsymbol{c}_y)$ such that $L(\boldsymbol{c}_y)\boldsymbol{c} \equiv \frac{1}{\chi(\boldsymbol{c}_y,\boldsymbol{c})}\mathcal{L}(\boldsymbol{c}_y)\boldsymbol{c}$. Following (6),

$$\mathcal{L}_{ij}(\boldsymbol{c}_y) = \begin{cases} \dfrac{\zeta^* - \zeta_y}{\zeta^* - \zeta_y^*}\left(1 - \dfrac{\zeta_y - \zeta_y^*}{\zeta^* - \zeta_y^*} c_{yi} c_{yj}^*\right) & i = j \\ -\dfrac{\zeta^* - \zeta_y}{\zeta^* - \zeta_y^*}\dfrac{\zeta_y - \zeta_y^*}{\zeta^* - \zeta_y^*} c_{yi} c_{yj}^* & i \neq j, \end{cases} \quad (9)$$

and $\chi(\boldsymbol{c}_y, \boldsymbol{c}) = |\mathcal{L}(\boldsymbol{c}_y)\boldsymbol{c}|$. If $\mathcal{L}(\boldsymbol{c}_y)$ is unitary, $L(\boldsymbol{c}_y) = \mathcal{L}(\boldsymbol{c}_y)$ and $L(\boldsymbol{c}_y)$ is linear. Thus, our problem reduces to the study of the linear transformations (9).

In subsection 3.1, we consider in detail logical operations on the two-qubit registers represented with four-component polarization vectors. However, they can be performed on many-qubit registers with a similar way, which is addressed in subsection 3.2.

3.1. Processing with two-qubit information

3.1.1. CNOT gate. The proposed implementation of the *CNOT* gate consists of two steps: rotating the polarization vector of the register and multiplying the resulting polarization by '-1'. Here, we focus on the first step addressing the second step at the end of subsection 3.1.3.

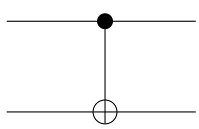
The rotation demands satisfaction of the condition $\zeta'' \ll \zeta_y''$ together with (i) $\zeta' = \zeta_y'$ or (ii) $|\zeta' - \zeta_y'| \ll \zeta_y''$. Following the first condition, our implementation of the gate is approximate. The computation accuracy depends on the ratio of the register to switching pulse widths; thus, it is determined by the method of creating the pulses and measuring their polarization. However, use of condition (ii) instead of (i) results in further loss of the computation accuracy. Condition (i) can be fulfilled for collisions of the SIT pulses while it is inapplicable to SF pulses since values $4\zeta', 4\zeta_y'$ are equal to the SF-soliton velocities following (2). Thus, if $\zeta' = \zeta_y'$, the relevant SF solitons cannot collide.

Taking the components of the polarization vector of the switching soliton $c_{yi} \equiv |c_{yi}|\,\mathrm{e}^{\mathrm{i}\varphi_{yi}}$ for $y = a$ such that

$$c_{a1} = c_{a2} = 0, \qquad |c_{a3}| = |c_{a4}| = \frac{1}{\sqrt{2}}, \qquad \varphi_{a3} - \varphi_{a4} = (2k+1)\pi, \quad (10)$$

where k is an integer, one finds

$$L(\boldsymbol{c}_a) \approx (-1)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \quad (11)$$

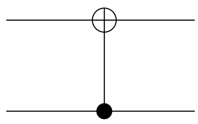


Up to the multiplier '−1', $L(\boldsymbol{c}_a)$ is the *CNOT* operator represented graphically as the quantum circuit above. The *CNOT* operation changing the qubit b can be performed with the switching soliton of the polarization $\boldsymbol{c}_b$:

$$c_{b1} = c_{b3} = 0, \qquad |c_{b2}| = |c_{b4}| = \tfrac{1}{\sqrt{2}}, \qquad \varphi_{b2} - \varphi_{b4} = (2k+1)\pi, \tag{12}$$

where k denotes an integer. The relevant transformation matrix of the register polarization is

$$L(\boldsymbol{c}_b) \approx (-1)\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\end{pmatrix}. \tag{13}$$

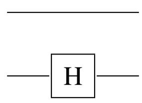


3.1.2. Hadamard gate. Since the Hadamard operation changes all the components of the two-qubit state vector and its matrix contains elements equal to zero, it cannot be performed with only one collision. In order to implement the *Hadamard* gate, we assemble two collisions of the register soliton with switching solitons of wavenumbers ζ_y, η_y and of polarizations $\boldsymbol{c}_y, \boldsymbol{d}_y$, respectively. Similarly, as for the *CNOT* gate, we assume $\zeta'' \ll \zeta_y'', \eta_y''$ together with (i) $\zeta' = \zeta_y' = \eta_y'$ or (ii) $|\zeta' - \zeta_y'| \ll \zeta_y''$, $|\zeta' - \eta_y'| \ll \eta_y''$. For the polarization vectors of the components ($c_{yj} = |c_{yj}|\,\mathrm{e}^{\mathrm{i}\varphi_{yj}}, d_{yj} = |d_{yj}|\,\mathrm{e}^{\mathrm{i}\phi_{yj}}$) given by

$$\begin{aligned} &c_{a1} = c_{a2} = d_{a3} = d_{a4} = 0, \qquad && |c_{a4}| = |d_{a2}| = \sqrt{\tfrac{1+\sqrt{2}}{2\sqrt{2}}},\\ &\varphi_{a3} - \varphi_{a4} = (2k+1)\pi, && \phi_{a1} - \phi_{a2} = (2l+1)\pi, \end{aligned} \tag{14}$$

the collisions transform the register as the *Hadamard* operation performed on the qubit a. The relevant transformation matrix takes the form

$$L(\boldsymbol{d}_a)L(\boldsymbol{c}_a) \approx \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1 & 0 & 0\\ 1 & -1 & 0 & 0\\ 0 & 0 & 1 & 1\\ 0 & 0 & 1 & -1\end{pmatrix}. \tag{15}$$

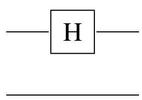


In order to perform the Hadamard operation on the qubit b, we use the switching solitons with

$$\begin{aligned} &c_{b1} = c_{b3} = d_{b2} = d_{b4} = 0, \qquad && |c_{b4}| = |d_{b3}| = \sqrt{\tfrac{1+\sqrt{2}}{2\sqrt{2}}},\\ &\varphi_{b2} - \varphi_{b4} = (2k+1)\pi, && \phi_{b1} - \phi_{b3} = (2l+1)\pi. \end{aligned} \tag{16}$$

The resulting state transformation corresponds to the matrix

$$L(\boldsymbol{d}_b)L(\boldsymbol{c}_b) \approx \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ 1 & 0 & -1 & 0\\ 0 & 1 & 0 & -1\end{pmatrix}. \tag{17}$$



3.1.3. $\pi/8$ gate and multiplication by a number. For the same reasons as mentioned considering the *Hadamard* gate implementation, the $\pi/8$ transformation and the multiplication by a number cannot be carried out with only one collision. This concerns the operation of the multiplication by a number even in the case when there is only one qubit in the register, as noted in [3]. Since the matrices of these operations are diagonal, they must be composed of the collisions with switching solitons with polarization vectors that have only one non-zero component. This property and the identity

$$\frac{\zeta^* - \zeta_y}{\zeta^* - \zeta_y^*} \equiv 1 - \frac{\zeta_y - \zeta_y^*}{\zeta^* - \zeta_y^*} \tag{18}$$

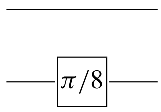
lead to diagonal and unitary transformation matrices $\mathcal{L}(\boldsymbol{c}_y)$ with identical diagonal elements except one. It enables performing the exact $\pi/8$ operation and the operations of the multiplication by a number via sequences of pulse collisions.

We perform the $\pi/8$ operation with two switching pulses whose wavevectors satisfy the conditions

$$\frac{\zeta^* - \zeta_y}{\zeta^* - \zeta_y^*} = \mathrm{e}^{\mathrm{i}\pi/4}, \qquad \frac{\zeta^* - \eta_y}{\zeta^* - \eta_y^*} = \mathrm{e}^{\mathrm{i}\pi/4}. \tag{19}$$

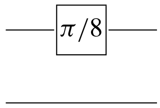
In particular, colliding the register with the pulses of the polarization-vector components $|c_{a4}| = |d_{a2}| = 1$, one achieves the $\pi/8$ transformation of the qubit a composed with the multiplication of the state vector by 'i',

$$L(\boldsymbol{d}_a)L(\boldsymbol{c}_a) = \mathrm{i}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\pi/4} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \mathrm{e}^{\mathrm{i}\pi/4} \end{pmatrix}. \tag{20}$$



The $\pi/8$ operation on the qubit b follows from the polarization components of the switching pulses $|c_{b4}| = |d_{b3}| = 1$. Then, the resulting register-transformation matrix takes the form

$$L(\boldsymbol{d}_b)L(\boldsymbol{c}_b) = \mathrm{i}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \mathrm{e}^{\mathrm{i}\pi/4} & 0 \\ 0 & 0 & 0 & \mathrm{e}^{\mathrm{i}\pi/4} \end{pmatrix}. \tag{21}$$



Following the construction of the $\pi/8$ gate, one can perform the multiplication of $\boldsymbol{c}$ by a factor $\mathrm{e}^{\mathrm{i}\varphi}$ with four collisions of the register soliton and switching solitons polarized along the different main axes. The wavenumbers ζ_y of these switching pulses have to satisfy

$$\frac{\zeta^* - \zeta_y}{\zeta^* - \zeta_y^*} = \mathrm{e}^{\mathrm{i}\varphi/5}. \tag{22}$$

The multiplication of the state vector by a number is a second step of the *CNOT* and $\pi/8$ operations. However, in order to reduce the number of soliton collisions, one can shift the multiplication operations to the end of the algorithm and, thus, perform only one multiplication.

3.2. Processing with n-qubit information

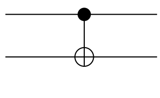
We will see that the construction of the universal set of quantum gates operating on two-qubit registers can be extended to many-qubit registers. Performing a gate operation on an n-qubit state demands use of 2^{n-2} times more collisions than for two qubits.

Let us study in detail the implementation of the *CNOT* operation increasing the number of qubits. We begin from adding a 'free' wire (a qubit unchanged via the gate action) at the bottom of the circuit (11). The gate is realized via two collisions of the eight-component register. The switching solitons of the polarizations $\boldsymbol{c}_b$, $\boldsymbol{d}_b$ such that

$$\begin{aligned} &|c_{b6}| = |c_{b8}| = |d_{b5}| = |d_{b7}| = 1/\sqrt{2}, \\ &\varphi_{b6} - \varphi_{b8} = (2k+1)\pi, \qquad \phi_{b5} - \phi_{b7} = (2l+1)\pi \end{aligned} \tag{23}$$

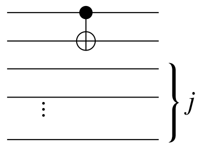
transform the register following the matrix

$$[CNOT]_1 \equiv L(\boldsymbol{d}_b)L(\boldsymbol{c}_b) = \begin{pmatrix} 1_2 & 0 & 0 & 0 \\ 0 & 1_2 & 0 & 0 \\ 0 & 0 & 0 & 1_2 \\ 0 & 0 & 1_2 & 0 \end{pmatrix}, \tag{24}$$



where 1_n denotes the unit matrix of rank n. Addition of j free wires at the bottom of the circuit (11) corresponds to the *CNOT* transformation represented as

$$[CNOT]_j = \begin{pmatrix} 1_{2^j} & 0 & 0 & 0 \\ 0 & 1_{2^j} & 0 & 0 \\ 0 & 0 & 0 & 1_{2^j} \\ 0 & 0 & 1_{2^j} & 0 \end{pmatrix}, \tag{25}$$



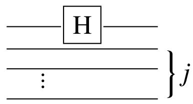
which can be performed with 2^j collisions of the register with the solitons of the linearly independent polarizations

$$\begin{aligned} \boldsymbol{c}_y &= [\underbrace{0, \ldots, 0}_{2^{j+2}-2^j-1}, \tfrac{1}{\sqrt{2}}\,\mathrm{e}^{\mathrm{i}\varphi_y}, \underbrace{0, \ldots, 0}_{2^j-1}, -\tfrac{1}{\sqrt{2}}\,\mathrm{e}^{\mathrm{i}\varphi_y}], \\ \boldsymbol{d}_y &= [\underbrace{0, \ldots, 0}_{2^{j+2}-2^j-2}, \tfrac{1}{\sqrt{2}}\,\mathrm{e}^{\mathrm{i}\phi_y}, \underbrace{0, \ldots, 0}_{2^j-1}, -\tfrac{1}{\sqrt{2}}\,\mathrm{e}^{\mathrm{i}\phi_y}, 0], \\ \boldsymbol{e}_y &= [\underbrace{0, \ldots, 0}_{2^{j+2}-2^j-3}, \tfrac{1}{\sqrt{2}}\,\mathrm{e}^{\mathrm{i}\psi_y}, \underbrace{0, \ldots, 0}_{2^j-1}, -\tfrac{1}{\sqrt{2}}\,\mathrm{e}^{\mathrm{i}\psi_y}, 0, 0], \\ &\vdots \end{aligned} \tag{26}$$

In the next step, we add m free wires at the top of the circuit. The relevant transformation matrix is block diagonal with 2^m identical $[CNOT]_j$ matrices on the diagonal. It can be decomposed into a product of 2^m block-diagonal matrices containing $2^m - 1$ unit blocks and one $[CNOT]_j$ block. All of the operations corresponding to the factor matrices can be performed independently with 2^j collisions since the polarizations of their switching solitons are linearly independent. The composition of these operations demands $2^{j+m} = 2^{n-2}$ collisions.

All the remaining universal gates operating on the two-qubit registers can be adapted to n-qubit circuits with similar consecutive addition of the free wires at the bottom and the top of the circuit, as was done above for the gate of (11). In particular, adding $j-1$ free wires to the bottom of the circuit of (17), one finds the operation

$$[H]_j = \frac{1}{\sqrt{2}}\begin{pmatrix} 1_{2^j} & 1_{2^j} \\ 1_{2^j} & -1_{2^j} \end{pmatrix} \tag{27}$$



which is realized via collisions with 2^j switching solitons of the polarizations

$$\begin{aligned} \boldsymbol{c}_y &= \Bigg[\underbrace{0, \ldots, 0}_{2^j-1}, \sqrt{\tfrac{\sqrt{2}-1}{2\sqrt{2}}}\,\mathrm{e}^{\mathrm{i}\varphi_y}, \underbrace{0, \ldots, 0}_{2^j-1}, -\sqrt{\tfrac{\sqrt{2}+1}{2\sqrt{2}}}\,\mathrm{e}^{\mathrm{i}\varphi_y}\Bigg], \\ \boldsymbol{d}_y &= \Bigg[\underbrace{0, \ldots, 0}_{2^j-2}, \sqrt{\tfrac{\sqrt{2}-1}{2\sqrt{2}}}\,\mathrm{e}^{\mathrm{i}\phi_y}, \underbrace{0, \ldots, 0}_{2^j-1}, -\sqrt{\tfrac{\sqrt{2}+1}{2\sqrt{2}}}\,\mathrm{e}^{\mathrm{i}\phi_y}, 0\Bigg], \\ \boldsymbol{e}_y &= \Bigg[\underbrace{0, \ldots, 0}_{2^j-3}, \sqrt{\tfrac{\sqrt{2}-1}{2\sqrt{2}}}\,\mathrm{e}^{\mathrm{i}\psi_y}, \underbrace{0, \ldots, 0}_{2^j-1}, -\sqrt{\tfrac{\sqrt{2}+1}{2\sqrt{2}}}\,\mathrm{e}^{\mathrm{i}\psi_y}, 0, 0\Bigg], \\ &\vdots \end{aligned} \tag{28}$$

Next, adding the free wires to the top of the circuit, one finds the transformation matrix to be block diagonal and factorizes the transformation into 2^{n-1} soliton collisions. The adaptation of $\pi/8$ and multiplication gates of subsection 3.1.3 to n-wire circuits is even more simple since all their switching solitons are polarized along the 2^n main axes. We omit its details. Let us only note that by multiplying an n-qubit state vector by $\mathrm{e}^{\mathrm{i}\varphi}$, we assume that the switching-soliton wavenumbers satisfy

$$\frac{\zeta^* - \zeta_y}{\zeta^* - \zeta_y^*} = \mathrm{e}^{\mathrm{i}\varphi/(2^n+1)} \tag{29}$$

instead of (22).

Colliding the register soliton with consecutive groups of switching solitons belonging to different gates enables one to carry out n-qubit quantum algorithms.

4. Conclusions

I have presented the concept of simulation of the quantum logic with vector solitons constructing the universal set of quantum gates. Its realization will enable one to simulate simple quantum circuits without using signal converters. However, the reduction of the number of elementary operations (soliton collisions) expected from using quantum algorithms instead of classical ones is not achieved since the number of collisions (and hence the time) necessary for realization of a single logical gate grows exponentially with the qubit number. This property and noise effects (increase of fluctuations of the soliton parameters with time [18]) limit the number of qubits and the circuit length (the number of gates). Similar restrictions are relevant to a simulation of the quantum logic with linear optics [19].

Usefulness of the vector solitons for quantum-communication purposes is limited following general limitations for the classical systems simulating quantum logic [17]. In spite of the fact that the register can be in an entangled state, the nonlocality characteristic of the quantum theory is absent. In particular, the soliton *CNOT* gate transforms any factorizable state $\boldsymbol{c}$, $(|\boldsymbol{c}\rangle = (x_b|0\rangle_b + y_b|1\rangle_b)(x_a|0\rangle_a + y_a|1\rangle_a))$, into an entangled but a local one $\boldsymbol{c}' = L[(0, 0, 1/\sqrt{2}, -1/\sqrt{2})]\boldsymbol{c} = (c_1, c_2, c_4, c_3)$, $(|\boldsymbol{c}'\rangle = x_b x_a|0\rangle_b|0\rangle_a + x_b y_a|0\rangle_b|1\rangle_a + y_b y_a|1\rangle_b|0\rangle_a + y_b x_a|1\rangle_b|1\rangle_a)$.

Two kinds of optical systems have been considered as media (displaying SF or SIT) for the pulse propagation. Use of the multi-component SIT has been found to enable *CNOT* and *Hadamard* gate simulation without the condition of different real parts of the colliding-soliton wavenumbers, which is impossible with the SF media. Because of this, the information processing with the SF pulses is potentially less accurate than with the SIT solitons. The possibility of using other media than those considered above is determined only by the rules of the pulse-polarization transformation due to the soliton collision (6) and, thus, by the form of the inverse scattering equations for relevant systems. They have to be similar to (4).

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